Standing Wave Solutions of Planar Irregular Hexagonal and Wye Resonators

JOSEPH HELSZAJN, MEMBER, IEEE

Abstract—**Suitable planar resonators for the design of three-port symmetrical junction circulators are the irregular hexagonal resonator and the wye resonator consisting of the junction of three open-circuited stubs. This paper describes the equipotential standing wave solutions and cutoff numbers of some lower order modes in such resonators using a finite element program. Circulator standing wave solutions in magnetized hexagonal and wye resonators are obtained by taking suitable combinations of those of the demagnetized resonators. The paper also includes the solution of planar resonators formed by the junction of four open-circuited stubs.**

I. Introduction

An IDEAL synthesis procedure for the design of planar junction circulators is one where the resonator shape can be varied or selected as part of the design [1]. To date, junction circulators using disk [2]–[5] and triangular [6], [7] resonators have been fully described. Two other planar resonators that have the symmetry of the three-port junction circulator are the irregular hexagonal resonator and the wye resonator formed by the junction of three open-circuited stubs. Although these two resonators are used commercially they have not been described in the literature. This paper gives the equipotential lines for the first three modes in each of these resonators using a finite element program [8], [9]. It also gives the cutoff numbers for each resonator. In the case of the irregular hexagonal resonator this is done as a function of the angle ϕ sustended by the smallest side of the hexagon. The case $\phi=0$ corresponds to a triangle and the case $\phi=60°$ describes a regular hexagon. The schematic diagrams of the two resonators described in this paper are depicted in Figs. 1 and 2. The cutoff number of the regular hexagon has also been calculated in [10] using a transverse resonance method.

Although it is difficult to visualize rotation of the equipotential lines in magnetized hexagonal and wye resonators it is nevertheless possible to construct circulation solutions using such resonators by taking suitable linear combinations of those of the demagnetized ones. This is done for the first two circulation modes of each resonator.

The paper also gives the standing wave patterns and cutoff numbers of the first six modes in planar X resonators formed by the junction of four open-circuited stubs.

Manuscript received October 26, 1979; revised December 23, 1980.
The author is with the Department of Electrical and Electronic Engineering, Heriot-Watt University, 31-35 Grassmarket, Edinburgh EH1 2HT, Scotland.

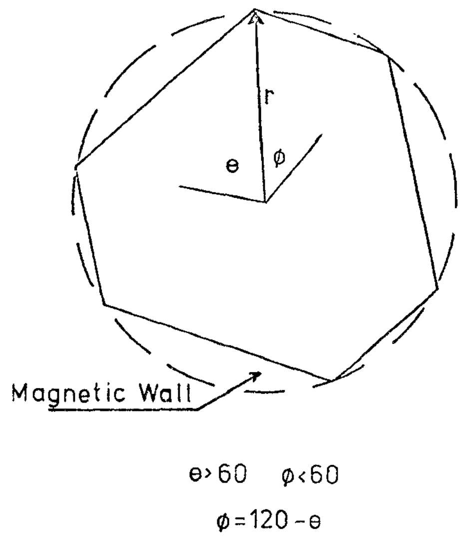

Fig. 1. Schematic diagram of irregular hexagonal resonator.

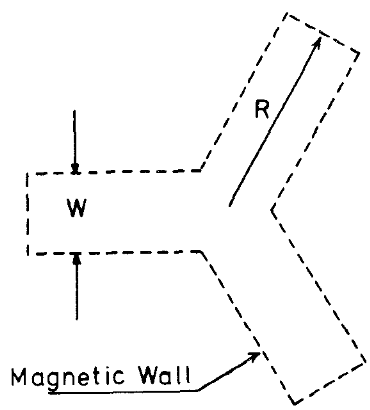

Fig. 2. Schematic diagram of wye resonator.

II. Equipotential Lines in Planar Irregular Hexagon Resonator

The equipotential lines and cutoff numbers of the first three modes of the irregular hexagon resonator in Fig. 1 have been computed using a finite element program package [8], [9] and are described in this section. The hexagon is subdivided into six triangular elements and a fourth order polynomial approximation is made to the EM fields in each triangle. A fourth order polynomial is used in the

0018-9480/81/0600-0562$00.75 © 1981 IEEE

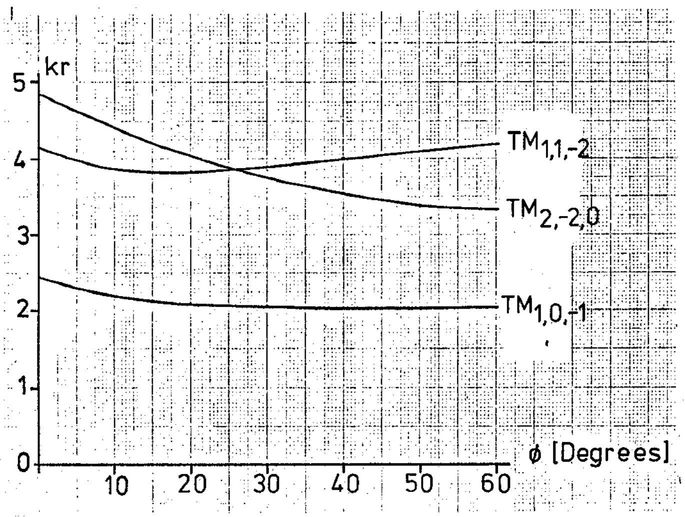

Fig. 3. Mode chart of irregular hexagonal resonator.

analysis in that the volume of labor involved in setting up the matrix problem for higher order polynomials becomes unwieldy. The number of triangles chosen is determined by the fact that the amount of computer time taken to solve the problem is not linearly dependent upon the number of triangles used [8].

A graph of the cutoff numbers of the first three modes versus the apex angle of the smaller triangle contained within the hexagon is illustrated in Fig. 3. The case where $\phi=0$ is a triangle and $\phi=60°$ is a regular hexagon. The modes of the irregular hexagon resonator are designated $TM_{m,n,L}$ limit modes. This nomenclature is consistent with that used in [6], [11] to describe the modes in triangular resonators. The meaning of m, n, L is defined in [11]. It is observed from Fig. 3 that the symmetric $TM_{1,1,-2}$ limit mode is degenerate with the $TM_{2,-2,0}$ limit mode. This property may find application in the design of junction circulators in which each eigenresonator is resonant [14].

The cutoff numbers for the first three regular hexagon resonators are

$$(kr)_{1,0,-1}=2.00$$
$$(kr)_{2,-2,0}=3.35$$
$$(kr)_{1,1,-2}=4.20$$

and for the triangular resonator

$$(kr)_{1,0,-1}=2.45$$
$$(kr)_{1,1,-2}=4.15$$
$$(kr)_{2,-2,0}=4.85$$

where

$$k=\frac{2\pi}{\lambda_0}\sqrt{\epsilon_f\mu_d}$$

ϵ_f is the relative dielectric constant, μ_d the relative demagnetized permeability of the ferrite material, and, λ_0 is the free space wavelength, in meters

$$m, n, l, \text{satisfy} \qquad m+n+l=0.$$

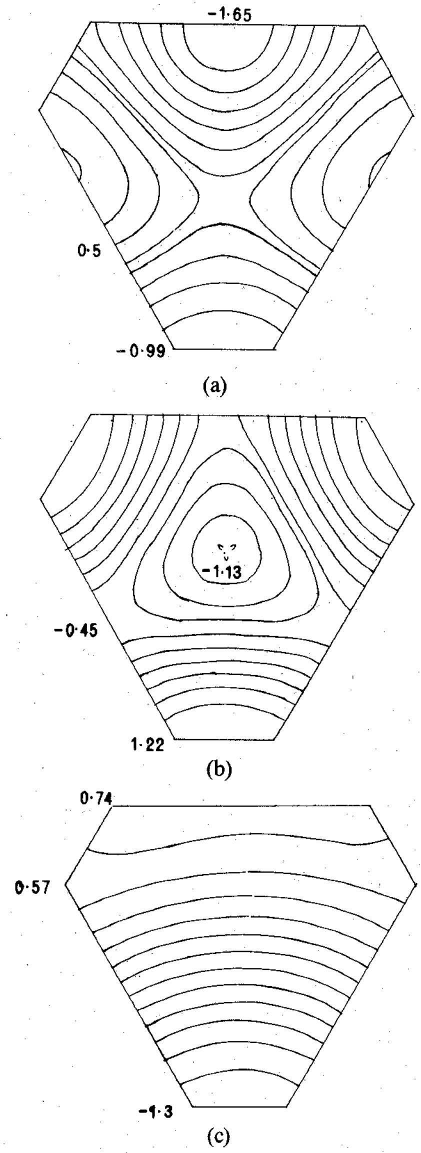

Fig. 4. Equipotential lines for: (a) $TM_{1,0,-1}$ limit mode in hexagonal resonator; (b) $TM_{2,-2,0}$ limit mode in hexagonal resonator; (c) $TM_{1,-1,-2,}$ limit mode in hexagonal resonator.

Fig. 4(a)–(c) indicates the equipotential lines of the first three TM modes in this resonator assuming perfect magnetic wall boundary conditions, exist at its periphery. In these illustrations ϕ is chosen as 30°. The lack of symmetry which may be noted in certain plots is due to the numerical approximation and can be eliminated by taking more elements in the finite element program.

The resonant modes produced by the program are orthogonal and they have been normalized so that the field distribution ϕ_a satisfy the condition

$$\int\int \phi_a^2 ds = 1.$$

It is observed that the standing wave solutions in Fig. 4(a)–(c) have the symmetry of demagnetized disk and triangular resonators used in circulator design [5], [6].

III. Equipotential Lines in Planar wye Resonator

The equipotential lines and cutoff numbers of the wye resonator in Fig. 2 have also been computed. [8], [9] The wye is subdivided into 13 triangular elements and a third order polynomial approximation is made to the EM fields in each triangle.

Fig. 5(a) indicates the equipotential lines of the fundamental TM mode in this resonator assuming that perfect magnetic wall boundary conditions exist at its periphery. This standing wave solution has the symmetry encountered in demagnetized disk and triangular resonators.

The cutoff number of this mode has also been computed and is given by

$$kr = 1.643$$

where

$$k = \frac{2\pi}{\lambda_0}\sqrt{\epsilon_f \mu_d}$$

ϵ_f is the relative dielectric constant, μ_d the relative demagnetized permeability of the ferrite material, and λ_0 is the free space wavelength, in meters. This cutoff number applies for $W/R = 0.4$, W and R defined in Fig. 2.

Fig. 5(b) and (c) depict the first symmetric and the first higher order TM modes in planar wye resonators. The cutoff numbers for these two modes are

$$kR = 3.33$$
$$kR = 4.91.$$

The equipotential lines of the symmetric mode in Fig. 5(b) displays a maximum value both at the center and at the end of the stub and exhibit a zero approximately midway along the stub at

$$kR \simeq 1.67.$$

The standing wave solution in Fig. 5(c) is also suitable for the construction of planar circulators.

IV. Standing Wave Solutions of Junction Circulators Using Hexagonal Resonators

It is observed that the equipotential lines in Fig. 4(a) and (b) have the symmetry encountered in demagnetized disk and triangular planar resonators used in circulator construction. They may therefore be used to construct circulators on planar magnetized circuits.

Although it is difficult to visualise rotation of the standing wave patterns within the resonator when it is magnetized, it is nevertheless possible to obtain phenomenological circulator boundary conditions with these resonators by taking a linear combination of two demagnetized field patterns [7]. This is shown in Fig. 6 for the dominant mode. This illustration suggests that an ideal circulation condition can be realized by coupling to the resonator at either terminals a or b.

The circulation solution in Fig. 6 is obtained by rotating the demagnetized standing wave pattern by 120° and subtracting it from the original solution [7].

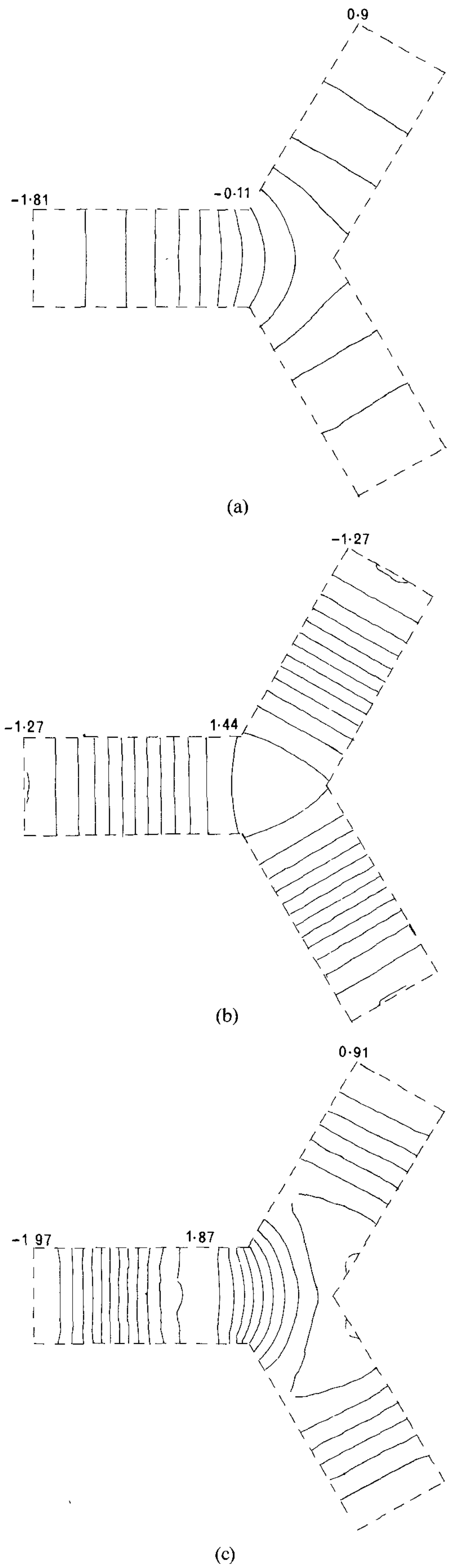

Fig. 5. Equipotential lines for: (a) dominant mode in wye resonator; (b) first symmetric mode in wye resonator; (c) first dominant mode in wye resonator.

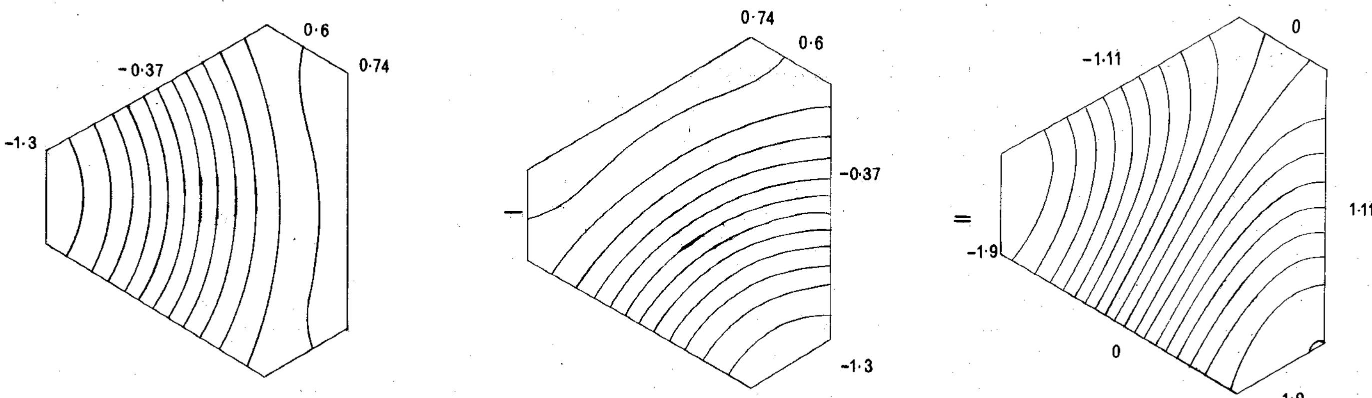

Fig. 6. Equipotential lines for dominant mode circulation solution in hexagonal resonator.

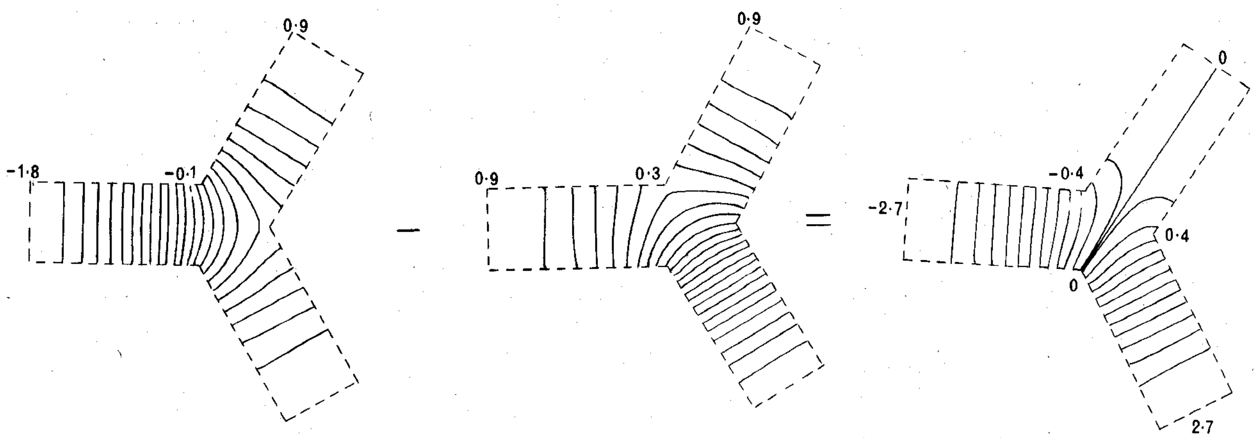

Fig. 7. Equipotential lines for dominant circulation mode in wye resonator.

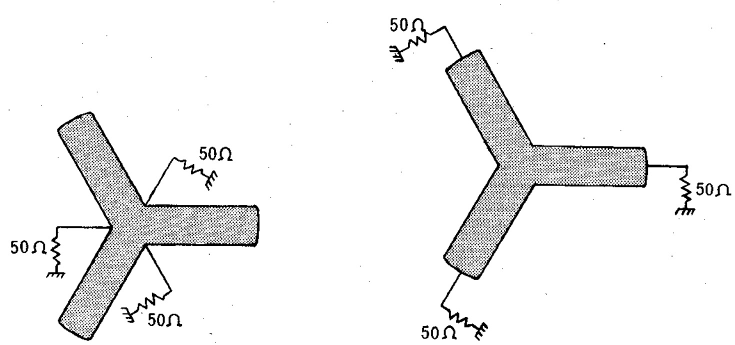

Fig. 8. Schematic diagram showing definition of: (a) "*aa*" terminals; (b) "*bb*" terminals.

V. Standing Wave Solution of Junction Circulators Using wye Resonators

The equipotential lines in Fig. 5(a) and (c) have also the symmetry encountered in planar resonators for use in the construction of planar circulators on magnetized substrates. Circulation solutions in magnetized wye resonators may therefore also be obtained by taking a linear combination of two standing wave solutions of the demagnetized wye resonator with one of them rotated through 120°. This is shown in Fig. 7 for the dominant mode in the wye resonator. Fig. 7 indicates that an ideal circulation condition can be realized by coupling to the wye resonator at either of the terminals illustrated in Fig. 8.

The arrangement in Fig. 8(a) leads to a widely used but ill understood commercial quarter-wave coupled three-port junction circulator whose outside radius is of the order of a quarter-wave at the operating frequency of the device. Fig. 9 indicates the frequency response of one commercial device based on this type of circuit.

Fig. 10 depicts the standing wave solution for the first higher order circulation mode in wye resonators.

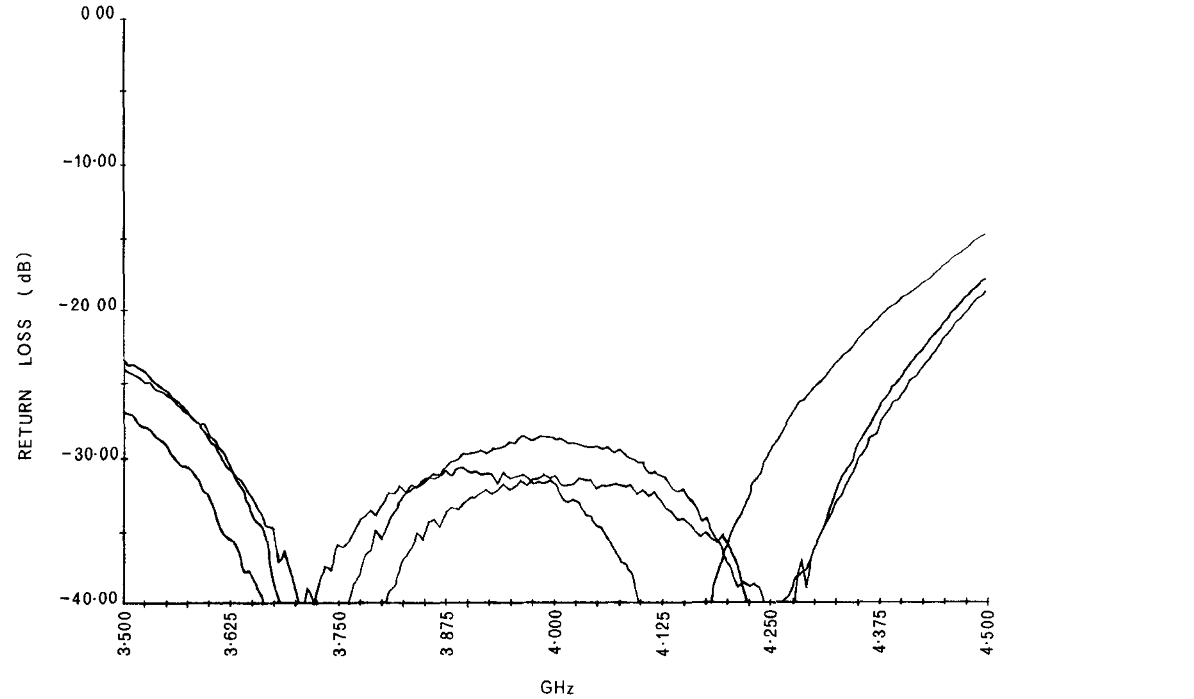

Fig. 9. Frequency response of quarter-wave coupled wye resonator.

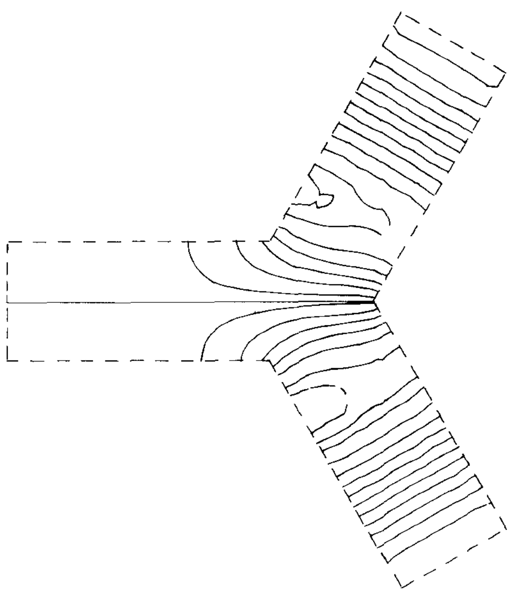

Fig. 10. Equipotential lines for the first higher order circulation mode in wye resonator.

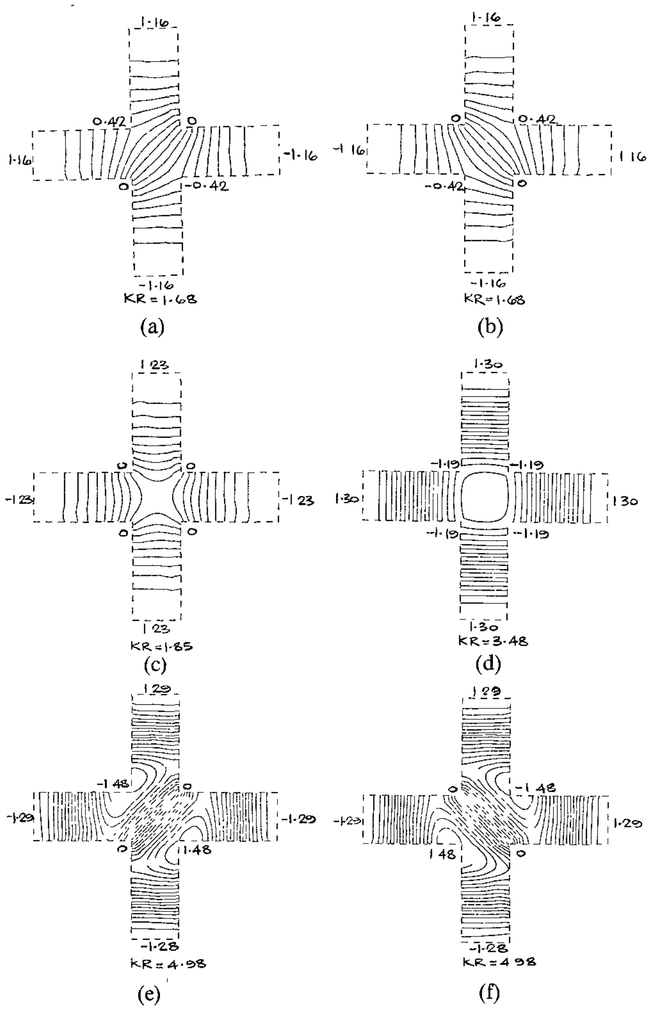

Fig. 11. Equipotential lines for: (a) and (b) dominant modes in X resonator; (c) first symmetric mode in X resonator; (d) second higher order mode in X resonator; (e) and (f) third higher order modes in X resonator.

VI. Standing Wave Solutions of Planar X Resonators

Symmetrical planar resonators may also be constructed by the symmetrical connection of any number of quarter-wave stubs. This section gives the equipotential lines and cutoff numbers of an X resonator formed by the junction of four open-circuited stubs. This resonator has the symmetry required for the construction of four-port single junction circulators.

Fig. 11(a)–(f) indicate the equipotential lines in the first six modes in the X resonator. The corresponding cutoff numbers are

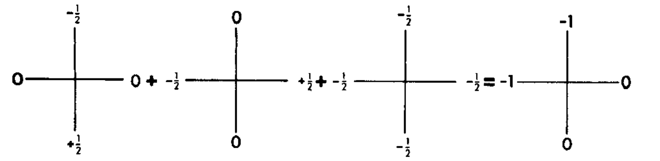

Fig. 12. Schematic diagram of mode patterns for the construction of four-port single-junction circulator.

$$kR=1.68$$
$$kR=1.68$$
$$kR=1.86$$
$$kR=3.48$$
$$kR=4.98$$
$$kR=4.98.$$

The standing wave solutions of the first three nonsymmetrical modes are formed by taking linear combinations of the intersection of two half-wave long stubs with appropriate open-circuited terminals. This is also the case for the first higher order mode except that now the intersecting stubs support one- and half-wave long resonances.

Fig. 12 depicts one possible linear combination of X resonator modes for the construction of a four-port single junction circulator. This adjustment may be realized experimentally by tuning the cutoff number of the first symmetric mode ($kR=3.48$) to that of the first nonsymmetric one ($kR=1.86$), with the aid of a thin metal post through the center of the resonator. Such a metal post will leave the nonsymmetric modes unperturbed since the electric field is zero at the center of the resonator for these modes.

An additional study of the X resonator indicates that the modes of this resonator are degenerate with those of a planar square resonator with similar magnetic side walls in the limit as the angle ϕ sustended at the origin by the width of the outside terminals approaches 90°. The modes of the X resonator may therefore be labelled $TM_{m,n,0}$ limit modes. For example, the two lowest order degenerate modes may be obtained by a superposition of the $TM_{1,0,0}$ and $TM_{0,1,0}$ modes in a planar square resonator.

VII. Conclusions

Two planar resonators having the symmetry required for the construction of three-port junction circulators are the irregular hexagon and the wye resonator. This paper has described the standing wave solutions and cutoff numbers for the first three modes in each of these resonators using a finite element program.

The standing wave solutions for the first two circulation modes in these resonators have also been obtained. Circulators using wye resonators lead to particularly compact quarter-wave coupled junction circulators. The paper includes the solution of some lower order modes in an X resonator formed by the symmetric junction of four open-circuited stubs.

Acknowledgment

The author would like to thank Dr. W. T. Nisbet and Dr. R. W. Lyon of Heriot-Watt University for the computations described in this paper.

References

[1] J. Helszajn, "Synthesis of quarter-wave and alternate-line transformer coupled microstrip circulators," to be published.

[2] U. Milano, J. H. Saunders, and L. Davis Jr., "A Y-junction stripline circulator," *IRE Trans. Microwave Theory Tech.*, vol. MTT-8, pp. 346–351, 1960.

[3] H. Bosma, "On stripline Y circulation at UHF," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-12, pp. 61–72, 1964.

[4] J. B. Davies and P. Cohen, "Theoretical design of symmetrical junction circulators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-11, pp. 506–512, 1963.

[5] C. E. Fay and R. L. Comstock, "Operation of the ferrite junction circulator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-13, pp. 15–27, Jan. 1965.

[6] J. Helszajn and D. S. James, "Planar triangular resonators with magnetic walls," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 95–100, 1978.

[7] J. Helszajn, D. S. James, and W. T. Nisbet, "Circulators using planar triangular resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 188–193, February 1979.

[8] P. Silvester, "High order polynomial triangular finite elements for potential problems," *Int. J. Eng. Sci.*, vol. 7 pp. 849–61, 1961.

[9] Z. J. Csendes and P. Silvester, "Numerical solution of dielectric loaded waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 1124–31, 1970.

[10] W. T. Nisbet and J. Helszajn, "Mode charts for microstrip resonators on dielectric and magnetic substrates using a transverse resonance method," *Microwave, Opt., Acoust.*, vol. 3, no. 2, pp. 69–77, Mar. 1979.

[11] S. A. Schelkunoff, *Electromagnetic Waves.* New York: Van Nostrand, 1943, p. 393.

[12] Y. Akaiwa, "Mode classification of a triangular ferrite post for Y-circulator operation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 59–61, Jan. 1977.

[13] W. Menzel, "Frequency dependent transmission properties of microstrip Y-junctions and 120° bends," *Microwaves, Opt., Acoust.*, vol. 2, no. 2, pp. 55–59, Mar. 1978.

[14] J. Helszajn, "Three-resonant-mode adjustment of the waveguide circulator," *Radio Electron. Eng.*, vol. 42, pp. 213–216, May 1972.